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A neutral spinning particle in interaction with a magnetic field and Poschl–Teller potential

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Abstract. The problem of a neutral spinning particle in interaction with a linear increasing rotating magnetic field and a Poschl–Teller potential is considered via path integrals. The calculations are carried out explicitly using an external current source. The problem is then reduced to that of a spinning forced Poschl–Teller oscillator whose spin is coupled to external derivative current sources. The result of the propagator is given as a series. The relative propagator of this forced oscillator is converted to that of an angular momentum via an extension of the dimension. Next, the series is exactly summed by means of a Laplace transformation and the orthonormalization relation of the eigenfunctions of the angular momentum.

1 Introduction

The Schrödinger equation has played an important role in quantum mechanics and is still the principal preoccupation of theoretical and experimental physics, because quantum mechanics has conquered modern physics via quantitative results. This equation is usually illustrated by some simple problems whose solutions are obtained in the analytical form to which physicists generally have recourse to illustrate the reality by some simple analytical models. In addition, we can also claim that this class of analytical models has been enlarged owing to the supersymmetry techniques based essentially on the idea of factorization. Consequently, the number of exactly solvable potentials has never ceased to increase and this list is still open. However, there exists in quantum mechanics a fundamental entity, spin, without which the explanation of numerous experiences is not effectively acceptable. This category of phenomena is described by the Pauli equation, which is an extension of the Schrödinger equation. The Pauli Hamiltonian contains besides the Schrödinger one a term describing the spin–field interaction. As a consequence, one would also search the general class of Pauli solvable problems, which would without doubt be profitable for applied physics. As an example, let us quote one case of this class which has become very popular owing to its direct application to the practical domain of physics. This is the well-known time dependent field (rotating) acting on a two level atom whose evolution is described by a Pauli type equation and which has made conspicuous the transition probabilities [1]. Without exaggeration, we can also say that, apart from this type of interaction and some ones relating to it [2], there exist few analytical and exact calculations treating the time dependent spin–field interaction. Moreover, if a space dependence of the exterior field occurs of the time dependent one this list becomes more restrictive [3]. In addition, the task will be almost unsolvable if we attempt to construct these solutions by path integral techniques, because as everyone knows the spin is an incontestable quantum physical quantity which takes only discrete values. This difficulty is tied to the fact that the path integral needs some conceptually classical objects like trajectories and clearly until now we do not know how to handle this technique in this important case. To this end, some attempts have been made to give a partial solution by using the Schwinger model of spin and according to these techniques a few explicit calculations are then readily carried out [4].

In this paper, which is a continuation of previous works [8,9], our aim is to partially solve this type of problems by considering the case of a neutral spinning particle moving in the increasing rotating space magnetic field and Poschl–Teller oscillator potential described by the following Hamiltonian:

$$H = \frac{p^2}{2m} + \frac{1}{2m}\left(c_1 \tan\alpha y + c_2 \cot\alpha y\right)^2 + \mu_0 \mathbf{B}\sigma, \tag{1}$$

where

$$\mathbf{B}(y) = B(y)\Big(\cos\left(2\kappa y + 2\delta(y)\right), 0, \sin\left(2\kappa y + 2\delta(y)\right)\Big), \tag{2}$$

$$B^2(y) = \left(\frac{\kappa}{m\mu_0}\right)^2 \left(c_1 \tan\alpha y + c_2 \cot\alpha y\right)^2 + \left(\frac{\alpha}{2m\mu_0}\right)^2 \left(c_1 \sec^2\alpha y - c_2 \csc^2\alpha y\right)^2 \tag{3}$$

and

$$\tan 2\delta(y) = \frac{2\kappa}{\alpha}\frac{c_1 \tan\alpha y + c_2 \cot\alpha y}{c_1 \sec^2\alpha y - c_2 \csc^2\alpha y}; \tag{4}$$

$\lambda = 1/(2m)$, $y \in [0, \pi/(2\alpha)]$, and α, c_1, and c_2, are positive constants.

This Hamiltonian describes the motion of a non-charged spinning particle moving along the y axis in interaction with a magnetic field rotating in the x–z plane with an angle depending on some function of the y variable. This has been solved first in [5] using the familiar factorization method for solving the Schrödinger equation. As is well known, this is intimately related to supersymmetric quantum mechanics and to our knowledge there has been no attempt in treating this using path integral techniques apart from works of Kubo [6] and of Junker [7] which are based respectively on semiclassical and quasi-semiclassical methods. We can assert that there is no exact calculus for this and the aim of this paper is to give a first attempt for the previous case where we will present an equivalent path integral method based on the technique of external current sources and an extension of dimension. In effect, the difficulty in treating this type of interaction resides in the fact that when one tries to introduce some rotations on the spin states which diagonalize the Hamiltonian, the non-homogeneity of the field gives rise to some corrections on the exterior motion as an effective potential and apparently this complicates the task. Now, in spite of the need of a general method, treating this type of interaction in the context of a path integral framework, there exists a method which tries to give a solution to this problem even in part [8–10].

In Sect. 2, we will give some notation and definitions needed for our further computations, and to deal with the problem we will follow exactly the same technique as of [9]. In Sect. 3, we restrict ourselves to the particular case of the neutral spinning particle in interaction with an increasing rotating magnetic field and the Poschl–Teller potential. For this case the series is exactly summed by converting the problem on the sphere via an extension of the dimension and using the orthogonalization relation of the eigenfunctions of angular momentum. The energy spectrum and the corresponding wave functions are deduced. Section 4 is devoted to our concluding remarks.

2 Formalism and method

In this section, we will first present our strategy in treating this type of problem by elaborating a general method relative to the Hamiltonian given by

$$H = \frac{p^2}{2m} + V(y) - \mu_0 B(y)\mathbf{n}(y)\sigma, \tag{5}$$

where we choose the orientation of the magnetic field

$$\mathbf{n}(y) = (\sin(2\kappa y + 2\delta(y)), 0, \cos(2\kappa y + 2\delta(y))), \tag{6}$$

and $\hat{V}(y), B(y)$ and $\delta(y)$ are arbitrary functions which would be judiciously chosen in the explicit application.

Now, let us focus on some definitions, properties and notation needed for the further developments. As we are interested in the spin–field interaction, we shall replace the Pauli matrices σ_i by a pair of fermionic operators (u, d) known as the Schwinger fermionic model of spin, following the recipe

$$\sigma_i \longrightarrow (u^+, d^+)\, \sigma_i \begin{pmatrix} u \\ d \end{pmatrix}, \tag{7}$$

where the pair (u, d) describes two-dimensional fermionic oscillators [4].

According to this replacement, the Hamiltonian converts to the following fermionic form:

$$H = \frac{p^2}{2m} + V(y) - \mu_0 B(y) \left(u^\dagger, d^\dagger\right) \mathbf{n}(y)\sigma \begin{pmatrix} u \\ d \end{pmatrix}. \tag{8}$$

Furthermore, it is suitable to take the quantum state as $| y, \eta\rangle$, where y describes the exterior evolution of the particle and η describes the spin dynamics. Following the habitual construction procedure of the path integral, we define the propagator as the matrix element of the evolution operator between the initial state $| y_a, \eta_a\rangle$ and the final state $| y_b, \eta_b\rangle$

$$K(y_b, \eta_b, y_a, \eta_a; T) = \langle y_b, \eta_b \mid \mathbf{U}(T) \mid y_a, \eta_a\rangle, \tag{9}$$

where

$$\mathbf{U}(T) = \mathbf{T}_D \exp\left(-\mathrm{i}\int_0^T H \mathrm{d}t\right), \tag{10}$$

where $\mathbf{T}_D$ is the Dyson time ordered operator, and next discretize the time T: $\epsilon = T/(N+1)$. The use of the Totter formula and the introduction at each intermediate instant of time of the resolution relations

$$\int \exp(-\eta^*\eta) \mid \eta\rangle\langle\eta \mid \mathrm{d}\eta\mathrm{d}\eta^* = 1 \tag{11}$$

and

$$\int \mid y\rangle\langle y \mid \mathrm{d}y = 1, \tag{12}$$

allow us to obtain the following discretized path integral form of the propagator:

$$\begin{aligned} K(y_b, \eta_b, y_a, \eta_a; T) &= \lim_{N\to\infty} \int \prod_{n=1}^{N+1} \left(\frac{m}{2\pi \mathrm{i}\epsilon}\right)^{1/2} \prod_{n=1}^{N} \left(\mathrm{d}y_n \mathrm{d}\eta_n \mathrm{d}\eta_n^* \mathrm{e}^{-\eta_n^*\eta_n}\right) \\ &\times \exp\left\{\mathrm{i}\sum_{n=1}^{N+1}\left[\frac{m}{2\epsilon}(y_n - y_{n-1})^2 - \epsilon V(y_n) - \mathrm{i}\eta_n^*\eta_{n-1} \right.\right. \\ &\left.\left. + \epsilon\mu_0 B(y_n)\eta_n^*\mathbf{n}(y_n)\sigma\eta_{n-1}\right]\right\}, \end{aligned} \tag{13}$$

with

$$y_0 = y_a, \quad y_{N+1} = y_b, \quad \eta_0 = \eta_a, \quad \text{and} \quad \eta_{N+1}^* = \eta_b^*. \tag{14}$$

This last expression represents the path integral of the propagator which has been the subject of our previous papers [4] and has the advantage of giving the chance to explicitly perform some concrete calculations.

Our strategy to treat this kind of problem was exposed in [9] and it consists of what follows. Knowing that integrations on the Grassmmanian variables are of Gaussian type their evaluation is immediate. With this intention, one initially introduces two external sources which enable us to quickly write the perturbation series by decoupling the diagonal terms from non-diagonal ones. Indeed, in this way the non-diagonal ones will contain a functional derivative relative to these currents, which in their turn will act on the diagonal ones. The latter are written in the form of a path integral relating to a particle subject to the action of a scalar potential forced by external currents.

After all these manipulations, it is easy to show that the calculation of the propagator series is reduced to

$$\begin{aligned} K(y_b,y_a;T) &= \mathrm{e}^{-\mathrm{i}(\kappa^2/2m)T}\mathrm{e}^{\mathrm{i}\sigma_y(\kappa y_b)} \\ &\times\left[K^{\mathcal{I}}(y_b,y_a;T)+\sum_{n=1}^{\infty}\left[\left(\prod_{j=1}^{n}\int_0^{t_{j-1}}\mathrm{d}t_j\int\mathrm{d}y_j\right)\right.\right. \\ &\times K^{\mathcal{I}}(y_b,y_1;T-t_1) \\ &\times\Lambda^{\mathcal{I}}(t_1)K^{\mathcal{I}}(y_1,y_2;t_1-t_2)\cdots\Lambda^{\mathcal{I}}(t_{n-1}) \\ &\times K^{\mathcal{I}}(y_{n-1},y_n;t_{n-1}-t_n) \\ &\left.\left.\times\,\Lambda^{\mathcal{I}}(t_n)K^{\mathcal{I}}(y_n,y_a;t_n-0)\right]\right]\Bigg|_{\mathcal{I}=\mathcal{I}^*=0}\mathrm{e}^{-\mathrm{i}\sigma_y(\kappa y_a)}, \end{aligned} \tag{15}$$

where the propagator $K^{\mathcal{I}}(y_b,y_a;T)$ is given by

$$\begin{aligned} K^{\mathcal{I}}(y_b,y_a;T) = \int\mathcal{D}y\exp\Bigg\{\mathrm{i}\int_0^T\mathrm{d}t\Bigg[&\frac{m}{2}\dot{y}^2-V(y) \\ &+(\mathrm{i}\kappa\dot{y}+\mu_0B(y)\sin2\delta(y))\,\mathcal{I}(t) \\ &+(-\mathrm{i}\kappa\dot{y}+\mu_0B(y)\sin2\delta(y))\,\mathcal{I}^*(t) \\ &+\mu_0B(y)\cos2\delta(y)\sigma_z\Bigg]\Bigg\} \end{aligned} \tag{16}$$

and

$$\Lambda^{\mathcal{I}}(t_n)=\begin{pmatrix}0 & \dfrac{\delta}{\delta\mathcal{I}(t_n)}\\ \dfrac{\delta}{\delta\mathcal{I}^*(t_n)} & 0\end{pmatrix}. \tag{17}$$

Now, we can say that the evaluation of the propagator of the system is reduced to the calculation of the propagator (16), which, in principle, will be easy to carry out. It is now readily seen that its matrix form is given by

$$K^{\mathcal{I}}(y_b,y_a;T)=\begin{pmatrix}K^{\mathcal{I}}_{+}(y_b,y_a;T) & 0\\ 0 & K^{\mathcal{I}}_{-}(y_b,y_a;T)\end{pmatrix}, \tag{18}$$

where

$$\begin{aligned} K^{\mathcal{I}}_{\pm}(y_b,y_a;T) = \int\mathcal{D}y\exp\Bigg\{\mathrm{i}\int_0^T\mathrm{d}t\Bigg[&\frac{m}{2}\dot{y}^2-V(y) \\ &+(\mathrm{i}\kappa\dot{y}+\mu_0B(y)\sin2\delta(y))\,\mathcal{I}(t) \\ &+(-\mathrm{i}\kappa\dot{y}+\mu_0B(y)\sin2\delta(y))\,\mathcal{I}^*(t) \\ &\pm\mu_0B(y)\cos2\delta(y)\Bigg]\Bigg\}. \end{aligned} \tag{19}$$

At this level it is necessary to pause because of the non-specified form of $V(y)$, $B(y)$ and $\delta(y)$. Then, in order to proceed in our calculations, we suppose that the expression of the previous propagator can be evaluated explicitly in a manner that allows us to write the spectral decomposition for this propagator. Next, taking advantage of the recurrence formula of wave functions corresponding to the problem related to the scalar potential $V(y)$ which is forced by the exterior sources and coupled to the rotating magnetic field, the action of $\Lambda^{\mathcal{I}}$ on $K^{\mathcal{I}}(n,n-1)$ shall easily be computed. Equalizing the exterior sources at zero and inserting the result into (15), the integration over $\{y\}$ would be computed using the orthonormalization properties. Finally, the integration over intermediate instants t_n will then be done using a Laplace transformation or other methods.

In the next section we will consider the special case given by the relations (1)–(4).

3 Application

Before beginning to treat the explicit application given by the relations (3) and (4), it is suitable to simplify $B(y)\cos2\delta(y)$ and $B(y)\sin2\delta(y)$ to the following expressions, where $y\in[0,\pi/(2\alpha)]$:

$$B(y)\sin2\delta(y)=\frac{\kappa}{m\mu_0}(c_1\tan\alpha y+c_2\cot\alpha y), \tag{20}$$

$$B(y)\cos2\delta(y)=\frac{\alpha}{2m\mu_0}\left(c_1/\cos^2\alpha y-c_2/\sin^2\alpha y\right) \tag{21}$$

After a substitution of these expressions in (19), the propagator $K^{\mathcal{I}}_{\pm}(y'',y';s''-s')$ becomes

$$\begin{aligned} K^{\mathcal{I}}_{\pm}(y_b,y_a;T) &= \int\mathcal{D}y(t)\exp\Bigg\{\mathrm{i}\int_0^T\Bigg[\frac{m}{2}\dot{y}^2 \\ &-\frac{\alpha^2}{2m}\left(\frac{c_1^{\pm}(c_1^{\pm}-1)}{\cos^2\alpha y}+\frac{c_2^{\pm}(c_2^{\pm}-1)}{\sin^2\alpha y}\right)+\frac{1}{2m}(c_1-c_2)^2 \\ &+\mathrm{i}\left(\kappa\dot{y}+\mathrm{i}\frac{\kappa}{m}(c_1\tan\alpha y+c_2\cot\alpha y)\right)\mathcal{I}(t) \\ &+\mathrm{i}\left(-\kappa\dot{y}+\mathrm{i}\frac{\kappa}{m}(c_1\tan\alpha y+c_2\cot\alpha y)\right)\mathcal{I}^*(t)\Bigg]\mathrm{d}t\Bigg\}, \end{aligned} \tag{22}$$

where $c_1^{\pm}$ and $c_2^{\pm}$ are defined by

$$c_1^{+}=\frac{c_1}{\alpha}+1,\quad c_2^{+}=\frac{c_2}{\alpha},\quad c_1^{-}=\frac{c_1}{\alpha}$$

and

$$c_2^{-}=\frac{c_2}{\alpha}+1. \tag{23}$$

It is remarkable that the previous propagator $K^{\mathcal{I}}_{\pm}(y_b,y_a;T)$ represents the one of the Poschl–Teller oscillator enforced by the external sources $\mathcal{I}(t)$ and $\mathcal{I}^*(t)$. Therefore, to give a solution to this problem we shall reduce it to a sphere variety by using the trick of the dimension extension. This fact allows the manifestation of the dynamical symmetry of the problem, thus simplifying the computations.

Therefore, we are firstly concerned with the calculation of the propagator for which the classical Lagrangian is given by

$$\begin{aligned}\mathcal{L}^{\mathcal{I}}_{\pm}(y,\dot{y};t) &= \frac{m}{2}\dot{y}^2 - \frac{\alpha^2}{2m}\left(\frac{c_1^{\pm}(c_1^{\pm}-1)}{\cos^2\alpha y} + \frac{c_2^{\pm}(c_2^{\pm}-1)}{\sin^2\alpha y}\right)\\ &\mathrm{i}\left(\kappa\dot{y} + \mathrm{i}\frac{\kappa}{m}(c_1\tan\alpha y + c_2\cot\alpha y)\right)\mathcal{I}(t)\\ &+\mathrm{i}\left(-\kappa\dot{y} + \mathrm{i}\frac{\kappa}{m}(c_1\tan\alpha y + c_2\cot\alpha y)\right)\mathcal{I}^*(t). \qquad (24)\end{aligned}$$

Defining the angular variable $\theta = 2\alpha y$ with $\theta \in [0,\pi]$ and using the following asymptotic formula:

$$\begin{aligned}&\exp\left[-\left(\frac{4p^2-\frac{1}{4}}{2z}\right)\right] \qquad (25)\\ &= \sqrt{\frac{z}{2\pi}}\int_0^{2\pi}\exp\left[\pm 2\mathrm{i}p\chi - z(1-\cos\chi)\right]\mathrm{d}\chi,\end{aligned}$$

which is valid for large z and $2p$ having a real value greater than unity [12].

We can write the action terms containing the $c_1^{\pm}$ and $c_2^{\pm}$ constants respectively as

$$\begin{aligned}&\exp\left(-\frac{\left(c_1^{\pm}-\frac{1}{2}\right)^2-\frac{1}{4}}{2\frac{m}{\mathrm{i}\epsilon\alpha^2}\cos^2(\theta_j/2)}\right) = \left[\frac{m}{2\pi\mathrm{i}\alpha^2\epsilon}\cos^2(\theta_j/2)\right]^{1/2}\\ &\times\int_0^{2\pi}\exp\Big[-\mathrm{i}\left(c_1^{\pm}-1/2\right)\chi_j + \left(\mathrm{i}m/\alpha^2\epsilon\right)\cos^2(\theta_j/2)\\ &\times(1-\cos\chi_j)\Big]\mathrm{d}\chi_j \qquad (26)\end{aligned}$$

and

$$\begin{aligned}&\exp\left(-\frac{\left(c_2^{\pm}-\frac{1}{2}\right)^2-\frac{1}{4}}{2\frac{m}{\mathrm{i}\epsilon\alpha^2}\sin^2(\theta_j/2)}\right) = \left[\frac{m}{2\pi\mathrm{i}\alpha^2\epsilon}\sin^2(\theta_j/2)\right]^{1/2}\\ &\times\int_0^{2\pi}\exp\Big[-\mathrm{i}\left(c_2^{\pm}-1/2\right)\beta_j + \left(\mathrm{i}m/\alpha^2\epsilon\right)\sin^2(\theta_j/2)\\ &\times(1-\cos\beta_j)\Big]\mathrm{d}\beta_j. \qquad (27)\end{aligned}$$

Let us notice that this latter technique, which we call the dimension extension, is a very powerful tool and has played an important role in the path integral. The substitution of these expressions in (22) gives the following result for the propagator:

$$\begin{aligned}K^{\mathcal{I}}_{\pm}(y_b,y_a;T) &= \frac{4}{\alpha}\left(\sin\theta'\sin\theta''\right)^{1/2}\int_0^{2\pi}\mathrm{d}\chi''\int_0^{2\pi}\mathrm{d}\beta''\\ &\times\lim_{N\to\infty}\int\prod_{j=1}^{N+1}\left[\frac{m}{8\pi\mathrm{i}\alpha^2\hbar\epsilon}\right]^{3/2}\\ &\times\prod_{j=1}^{N}\left(2\sin\theta_j\mathrm{d}\theta_j\mathrm{d}\chi_j\mathrm{d}\beta_j\right)\exp\frac{\mathrm{i}}{\hbar}\sum_{j=1}^{N+1}\left(\mathcal{S}^{\mathcal{I}}_{\pm j}\right), \qquad (28)\end{aligned}$$

where $\mathcal{S}^{\mathcal{I}}_{\pm j}$ is the action defined by

$$\begin{aligned}\mathcal{S}^{\mathcal{I}}_{\pm j} &= \frac{m}{8\alpha^2\epsilon}\left(\Delta\theta_j\right)^2 + \left(c_1^{\pm}-1/2\right)\chi_j + \left(c_2^{\pm}-1/2\right)\beta_j\\ &+\left(m/\alpha^2\epsilon\right)\cos^2(\theta_j/2)\left(1-\cos\chi_j\right)\\ &+\left(m/\alpha^2\epsilon\right)\sin^2(\theta_j/2)\left(1-\cos\beta_j\right)\\ &+\mathrm{i}\left(\frac{\kappa}{2\alpha}\Delta\theta_j + \mathrm{i}\epsilon\frac{\kappa}{m}\left(c_1\tan\frac{\theta_j}{2} + c_2\cot\frac{\theta_j}{2}\right)\right)\mathcal{I}_j\\ &+\frac{1}{2m}(c_1-c_2)^2 \qquad (29)\\ &-\mathrm{i}\left(\frac{\kappa}{2\alpha}\Delta\theta_j - \mathrm{i}\epsilon\frac{\kappa}{m}\left(c_1\tan\frac{\theta_j}{2} + c_2\cot\frac{\theta_j}{2}\right)\right)\mathcal{I}^*_j.\end{aligned}$$

At this stage, instead of the variables χ_j and β_j, we introduce the two Euler angular variables ψ_j and φ_j causing the following change:

$$\chi_j = \frac{1}{2}\left(\Delta\psi_j + \Delta\varphi_j\right); \quad \beta_j = \frac{1}{2}\left(\Delta\psi_j - \Delta\varphi_j\right), \qquad (30)$$

with $\varphi_j \in [0,2\pi]$ and $\psi_j \in [0,4\pi]$.

As is easy to verify, the path integral measure and the infinitesimal action become

$$\prod_{j=1}^{N}\left(\mathrm{d}\chi_j\mathrm{d}\beta_j\right) = \prod_{j=1}^{N}\left(\frac{1}{2}\mathrm{d}\varphi_j\mathrm{d}\psi_j\right) \qquad (31)$$

and

$$\begin{aligned}&\sum_{j=1}^{N+1}\left(c_1^{\pm}-1/2\right)\chi_j + \left(c_2^{\pm}-1/2\right)\beta_j\\ &= \left(c_1^{\pm}+c_2^{\pm}-1\right)\Big/2\sum_{j=1}^{N+1}\Delta\psi_j + \left(c_1^{\pm}-c_2^{\pm}\right)\Big/2\sum_{j=1}^{N+1}\Delta\varphi_j\\ &= \frac{1}{2}\left(c_1^{\pm}+c_2^{\pm}-1\right)\left(\psi_{N+1}-\psi_0\right)\\ &+\frac{1}{2}\left(c_1^{\pm}-c_2^{\pm}\right)\left(\varphi_{N+1}-\varphi_0\right). \qquad (32)\end{aligned}$$

Consequently, the path integral (16) takes the following form:

$$\begin{aligned}K^{\mathcal{I}}(y_b,y_a;T) &= \frac{\alpha}{8\pi^2}\left(\sin\theta_a\sin\theta_b\right)^{1/2}\\ &\times\int_0^{2\pi}\mathrm{d}\varphi_b\int_0^{4\pi}\mathrm{d}\psi_b\mathcal{K}^{\mathcal{I}}(\theta_b,\varphi_b,\psi_b,\theta_a,0,0;T)\\ &\times\mathrm{e}^{+(\mathrm{i}/2m)(c_1-c_2)^2-(\mathrm{i}/2\alpha)(c_1+c_2)(\psi_b-\psi_a)-(\mathrm{i}/2\alpha)(c_1-c_2)(\varphi_b-\varphi_a)}\\ &\times\begin{pmatrix}\mathrm{e}^{-(\mathrm{i}/2)(\varphi_b-\varphi_a)} & 0\\ 0 & \mathrm{e}^{+(\mathrm{i}/2)(\varphi_b-\varphi_a)}\end{pmatrix}, \qquad (33)\end{aligned}$$

where the kernel $\mathcal{K}^{\mathcal{I}}(\theta_b,\varphi_b,\psi_b,\theta_a,0,0;T)$ has the following path integral expression:

$$\begin{aligned}&\mathcal{K}^{\mathcal{I}}(\theta_b,\varphi_b,\psi_b,\theta_a,0,0;T)\\&=\lim_{N\to\infty}\int_0^{\pi}\int_0^{2\pi}\int_0^{4\pi}\prod_{j=1}^{N+1}\left[\frac{m}{8\pi\mathrm{i}\alpha^2\hbar\epsilon}\right]^{3/2}\\&\times\prod_{j=1}^{N}\left(\sin\theta_j\mathrm{d}\theta_j\mathrm{d}\varphi_j\mathrm{d}\psi_j\right)16\pi^2\exp\left(\frac{\mathrm{i}}{\hbar}\sum_{j=1}^{N+1}\mathcal{S}_j^{\mathcal{I}}\right),\end{aligned}\tag{34}$$

with

$$\begin{aligned}\mathcal{S}_j^{\mathcal{I}}&=\frac{m}{8\alpha^2\epsilon}\left(\Delta\theta_j\right)^2\\&+\left(m/\alpha^2\epsilon\right)\cos^2(\theta_j/2)\left(1-\cos(\Delta\psi_j+\Delta\varphi_j)/2\right)\\&+\left(m/\alpha^2\epsilon\right)\sin^2(\theta_j/2)\left(1-\cos(\Delta\psi_j-\Delta\varphi_j)/2\right)\\&+\mathrm{i}\Bigg(\frac{\kappa}{2\alpha}\Delta\theta_j+\mathrm{i}\epsilon\frac{\kappa}{m}\\&\times\left(\left(c_1+c_2\right)\frac{1}{\sin\theta_j}+\left(c_2-c_1\right)\cot\theta_j\right)\Bigg)\mathcal{I}_j\\&-\mathrm{i}\Bigg(\frac{\kappa}{2\alpha}\Delta\theta_j-\mathrm{i}\epsilon\frac{\kappa}{m}\\&\times\left(\left(c_1+c_2\right)\frac{1}{\sin\theta_j}+\left(c_2-c_1\right)\cot\theta_j\right)\Bigg)\mathcal{I}_j^{*}.\end{aligned}\tag{35}$$

Now, the use of the expansion of $\cos\Delta x_j$ at fourth order in Δx_j, allows us to write

$$\begin{aligned}&\cos(\Delta\psi_j+\Delta\varphi_j)/2\\&=1-\frac{1}{2^3}\left((\Delta\psi_j)^2+(\Delta\varphi_j)^2+2\Delta\psi_j\Delta\varphi_j\right)\\&+\frac{1}{2^4 4!}\left((\Delta\psi_j)^4+(\Delta\varphi_j)^4+6(\Delta\psi_j)^2(\Delta\varphi_j)^2\right)\end{aligned}\tag{36}$$

and

$$\begin{aligned}&\cos(\Delta\psi_j-\Delta\varphi_j)/2\\&=1-\frac{1}{2^3}\left((\Delta\psi_j)^2+(\Delta\varphi_j)^2-2\Delta\psi_j\Delta\varphi_j\right)\\&+\frac{1}{2^4 4!}\left((\Delta\psi_j)^4+(\Delta\varphi_j)^4+6(\Delta\psi_j)^2(\Delta\varphi_j)^2\right).\end{aligned}\tag{37}$$

As usually done in path integral techniques, we approximate $(\Delta\psi_j)^4$,$(\Delta\varphi_j)^4$, and $(\Delta\psi_j)^2.(\Delta\varphi_j)^2$ by quantum potential corrections following the standard procedure [11],

$$\begin{cases}\langle(\Delta\psi_j)^4\rangle=\langle(\Delta\varphi_j)^4\rangle\simeq 3\left(\dfrac{4\alpha^2\epsilon}{m}\right)^2,\\[2mm]\langle(\Delta\psi_j)^2(\Delta\varphi_j)^2\rangle\simeq\left(\dfrac{4\alpha^2\epsilon}{m}\right)^2.\end{cases}\tag{38}$$

Thus, the infinitesimal action $\mathcal{S}_j^{\mathcal{I}}$ turns into

$$\begin{aligned}\mathcal{S}_j^{\mathcal{I}}&=\frac{m}{8\alpha^2\epsilon}(\Delta\theta_j)^2+\left(m/8\alpha^2\epsilon\right)\left((\Delta\psi_j)^2+(\Delta\varphi_j)^2\right)\\&+\left(m/4\alpha^2\epsilon\right)\cos(\theta_j)\Delta\psi_j\Delta\varphi_j\\&-\frac{\alpha^2\epsilon}{2m}+\mathrm{i}\Bigg[(\kappa/2\alpha)\Delta\theta_j\\&+\mathrm{i}\epsilon\frac{\kappa}{m}\left(\left(c_1+c_2\right)\frac{1}{\sin\theta_j}+\left(c_2-c_1\right)\cot\theta_j\right)\Bigg]\mathcal{I}_j\\&-\mathrm{i}\Bigg[(\kappa/2\alpha)\Delta\theta_j-\mathrm{i}\epsilon\frac{\kappa}{m}\\&\times\left(\left(c_1+c_2\right)\frac{1}{\sin\theta_j}+\left(c_2-c_1\right)\cot\theta_j\right)\Bigg]\mathcal{I}_j^{*}.\end{aligned}\tag{39}$$

The evaluation of the propagator $\mathcal{K}^{\mathcal{I}}(\theta_b,\varphi_b,\psi_b,\theta_a,0,0;T)$ in the configuration space seems to be a very hard task due to the presence of the external sources. To overcome this difficulty, it is convenient to use the phase space by linearizing the quadratic terms of the infinitesimal action $(\Delta\theta_j)^2,(\Delta\psi_j)^2,(\Delta\varphi_j)^2$ following the formula

$$\int_{-\infty}^{+\infty}\exp\left(ap^2+bp\right)\mathrm{d}p=\sqrt{-\frac{\pi}{a}}\exp\left(-\frac{b^2}{4a}\right).\tag{40}$$

We will then get for the propagator $\mathcal{K}^{\mathcal{I}}(\theta_b,\varphi_b,\psi_b,\theta_a,0,0;T)$ the following expression:

$$\begin{aligned}&\mathcal{K}^{\mathcal{I}}(\theta_b,\varphi_b,\psi_b,\theta_a,0,0;T)\\&=16\pi^2\int_0^{\pi}\mathcal{D}\theta\sin\theta\int_0^{2\pi}\mathcal{D}\varphi\int_0^{4\pi}\mathcal{D}\psi\int\frac{\mathcal{D}p_\theta}{2\pi\hbar}\\&\times\int\frac{\mathcal{D}p_\varphi}{2\pi\hbar}\int\frac{\mathcal{D}p_\psi}{2\pi\hbar}\mathrm{e}^{-\mathrm{i}(\alpha^2/2m)T}.\\&\times\exp\mathrm{i}\int_{s'}^{s''}\left[p_\theta\dot\theta+p_\varphi\dot\varphi+p_\psi\dot\psi-H^{\mathcal{I}}\right]\mathrm{d}t,\end{aligned}\tag{41}$$

with

$$\begin{aligned}H^{\mathcal{I}}=&\frac{2\alpha^2}{m}\Bigg[p_\theta^2+\frac{1}{\sin^2\theta}\left(p_\varphi^2+p_\psi^2-2p_\varphi p_\psi\cos\theta\right)\\&+\frac{\kappa\hbar}{\alpha}\left(\cot\theta p_\varphi-\mathrm{i}p_\theta-\frac{p_\psi}{\sin\theta}\right)\mathcal{I}(t)\\&+\frac{\kappa\hbar}{\alpha}\left(\cot\theta p_\varphi+\mathrm{i}p_\theta-\frac{p_\psi}{\sin\theta}\right)\mathcal{I}^*(t)\Bigg].\end{aligned}\tag{42}$$

The latter formulae have been improved by extending the variables φ and ψ to the domain $(-\infty,+\infty)$ using the periodic replacement $(\varphi,\psi)\to(\varphi+2\pi N,\psi+2\pi N')$, then changing them as follows:

$$\begin{aligned}\varphi(t)&\to\varphi(t)+\frac{2\alpha\kappa}{m}\int_0^t\left(\mathcal{I}(s)+\mathcal{I}^*(s)\right)\cot\theta(s)\mathrm{d}s,\\\psi(t)&\to\psi(t)-\frac{2\alpha\kappa}{m}\int_0^t\left(\mathcal{I}(s)+\mathcal{I}^*(s)\right)\frac{1}{\sin\theta(s)}\mathrm{d}s,\end{aligned}\tag{43}$$

and finally by omitting all the nonlinear terms in $\mathcal{I}(s)$ and $\mathcal{I}^*(s)$ which vanish because the matrix $\Lambda^{\mathcal{I}}(t_n)$ is linear in the derivative over the sources.

Furthermore, it is easy to show that the Hamiltonian $H^{\mathcal{I}}$ is closely related to the one of the top with a momentum $\mathbf{J}$. In effect, this correspondence will be visible if we put

$$\left.\begin{aligned} J_1 &= -p_\varphi \sin\varphi \cot\theta + p_\theta \cos\varphi + p_\psi \frac{\sin\varphi}{\sin\theta}, \\ J_2 &= p_\varphi \cos\varphi \cot\theta + p_\theta \sin\varphi - p_\psi \frac{\cos\varphi}{\sin\theta}, \\ J_3 &= p_\varphi, \end{aligned}\right\} \tag{44}$$

where J_1,J_2 and J_3 are the components of the momentum $\mathbf{J}$ verifying the following Poisson brackets:

$$\{J_i, J_j\} = J_k, \quad i, j, k \text{ is a cyclic permutation of } 1, 2, 3. \tag{45}$$

This result is a consequence of the canonical Poisson brackets of the phase space variables $(p_\theta, p_\varphi, p_\psi, \theta, \varphi, \psi)$.

Thus, the Hamiltonian $H^{\mathcal{I}}$ will be written as

$$H^{\mathcal{I}} = \frac{2\alpha^2}{m}\left[\mathbf{J}^2 + (\kappa/\alpha)\,\mathrm{e}^{-\mathrm{i}\varphi}\mathcal{I}(t)\mathbf{J}_+ + (\kappa/\alpha)\mathrm{e}^{+\mathrm{i}\varphi}\mathcal{I}^*(t)\mathbf{J}_-\right], \tag{46}$$

with

$$\mathbf{J}_\pm = J_1 \pm \mathrm{i}J_2.$$

The components $\mathbf{J}_\pm$, being operators, are the expression, extended to the sphere, of the operators $L_\pm$ of [5].

Now, the appearance of $\mathrm{e}^{-\mathrm{i}\varphi}$ and $\mathrm{e}^{+\mathrm{i}\varphi}$ besides respectively the sources $\mathcal{I}(t)$ and $\mathcal{I}^*(t)$ constrain a little more the computations and in order to reduce the problem again, let us introduce the following replacement:

$$(\kappa/\alpha)\mathrm{e}^{-\mathrm{i}\varphi}\mathcal{I}(t) = \Upsilon(t), \quad (\kappa/\alpha)\mathrm{e}^{+\mathrm{i}\varphi}\mathcal{I}^*(t) = \Upsilon^*(t), \tag{47}$$

which gives for $H^{\mathcal{I}}$ the following form:

$$H^{\Upsilon} = \frac{2\alpha^2}{m}\left[\mathbf{J}^2 + \Upsilon(t)\mathbf{J}_+ + \Upsilon^*(t)\mathbf{J}_-\right] \tag{48}$$

and for the differential matrix $\Lambda^{\mathcal{I}}(t_n)$ the following the expression:

$$\Lambda^{\mathcal{I}}(t_n) = \frac{\kappa}{\alpha}\begin{pmatrix} \mathrm{e}^{-(\mathrm{i}/2)\varphi_n} & 0 \\ 0 & \mathrm{e}^{+(\mathrm{i}/2)\varphi_n}\end{pmatrix}\Lambda^{\Upsilon}(t_n) \times \begin{pmatrix} \mathrm{e}^{+(\mathrm{i}/2)\varphi_n} & 0 \\ 0 & \mathrm{e}^{-(\mathrm{i}/2)\varphi_n}\end{pmatrix}, \tag{49}$$

with

$$\Lambda^{\Upsilon}(t_n) = \begin{pmatrix} 0 & \dfrac{\delta}{\delta\Upsilon(t_n)} \\ \dfrac{\delta}{\delta\Upsilon^*(t_n)} & 0 \end{pmatrix}. \tag{50}$$

At this level, let us to note that the computation of the propagator relative to the Hamiltonian (48) is readily done and the result is a generalization of that of [13] because of the presence of the term $\mathbf{J}^2$ in the Hamiltonian. In fact, the term containing the currents $\Upsilon(t)$ and $\Upsilon^*(t)$ can be treated as a perturbation. Accordingly, it is easy to show that the propagator $\mathcal{K}^{\Upsilon}(\theta_b, \varphi_b, \psi_b, \theta_a, 0, 0; T)$ will be given by the following expression projected on the spin states:

$$\begin{aligned}&\mathcal{K}^{\Upsilon}(\theta_b, \varphi_b, \psi_b, \theta_a, 0, 0; T) \\ &= 16\pi^2 \sum_{2J=0}^{+\infty}\ \sum_{m',m''=-J}^{+J}\ \sum_{\nu=-J}^{+J}\Big[\mathrm{e}^{-\mathrm{i}(\alpha^2/2m)(2J+1)^2 T} \\ &\times I^{\Upsilon}(m'', m'; T)\,(\kappa/\alpha)\,(2J+1)\,\mathrm{e}^{\mathrm{i}m''\varphi_b}\mathrm{e}^{\mathrm{i}\nu\psi_b}\mathrm{e}^{-\mathrm{i}m'\varphi_a} \\ &\times \mathrm{e}^{-\mathrm{i}\nu\psi_a} d^J_{m'',\nu}(\theta_b) d^J_{m',\nu}(\theta_a)\Big],\end{aligned} \tag{51}$$

where $d^J_{m,\nu}(\theta)$ is the Wigner function and we notice that this result without $I^{\Upsilon}(m'', m'; T)$ is nothing but the propagator of the free particle on the sphere [12].

The amplitude $I^{\Upsilon}(m'', m'; T)$ is given by [13]

$$\begin{aligned}&I^{\Upsilon}(m'', m'; T) \\ &= \sum_{l=0}^{\infty}\Bigg\{(-1)^l \frac{\sqrt{(J+m'')!\,(J-m'')!\,(J+m')!\,(J-m')!}}{l!\,(J-m''-l)!\,(J+m'-l)!\,(m''-m'+l)!} \\ &\times (f(T))^{J+m'-l}\,(g(T))^{m''-m'+l} \\ &\times \left(\bar{f}(T)\right)^{J-m''-l}(\bar{g}(T))^l\Bigg\}\end{aligned} \tag{52}$$

and f and g are complex functions verifying the following coupled differential equations

$$\frac{\mathrm{d}f}{\mathrm{d}s} = \mathrm{i}(2\alpha^2/m)\Upsilon\bar{g}, \quad \frac{\mathrm{d}g}{\mathrm{d}s} = -\mathrm{i}(2\alpha^2/m)\Upsilon\bar{f}, \tag{53}$$

with the following boundary conditions

$$f(0) = 1, \quad g(0) = 0. \tag{54}$$

In addition, in what follows we are not interested in the explicit solutions of these equations, but in their formal solutions which are given by

$$f(s) = \mathrm{i}(2\alpha^2/m)\int_0^s \Upsilon(\tau)\bar{g}(\tau)\mathrm{d}\tau + 1, \tag{55}$$

$$g(s) = -\mathrm{i}(2\alpha^2/m)\int_0^s \Upsilon(\tau)\bar{f}(\tau)\mathrm{d}\tau, \tag{56}$$

because it is the variational derivative over the external sources $\Upsilon(\tau)$ and $\Upsilon^*(\tau)$ which appears in the computations. In effect, we will only need the following derivative expressions:

$$\left.\frac{\delta g(s_{2j-1})}{\delta\Upsilon(s_{2j-1})}\right|_{\Upsilon,\Upsilon^*=0} = -\mathrm{i}(2\alpha^2/m),$$

and

$$\left.\frac{\delta\bar{g}(s_{2j})}{\delta\Upsilon^*(s_{2j})}\right|_{\Upsilon,\Upsilon^*=0} = +\mathrm{i}(2\alpha^2/m), \tag{57}$$

and all other derivatives vanish. This result is obtained with the help of the boundary conditions (54).

After this, let us turn to the evaluation of the perturbation series (15). Firstly, from the substitution of the result (51) in (33) we get

$$\begin{aligned}
&K^{\Upsilon}(y_n,y_{n+1};t_n-t_{n-1})\\
&=2\alpha e^{-(i/2m)\left(\alpha^2(2J+1)^2-(c_1-c_2)^2\right)T}\\
&\times(\kappa/\alpha)\left(\sin\theta_n\sin\theta_{n+1}\right)^{1/2}\\
&\times\int_0^{2\pi}d\varphi_n\int_0^{4\pi}d\psi_n\sum_{2J=0}^{+\infty}\sum_{m'_n,m''_n=-J}^{+J}\sum_{\nu=-J}^{+J}\\
&\times\Bigg[(2J+1)\,I^{\Upsilon}\left(m''_n,m'_n;t_n-t_{n-1}\right)e^{im''_n\varphi_n}\\
&\times e^{i\nu\psi_n}e^{-im'_n\varphi_{n+1}}e^{-i\nu\psi_{n+1}}d^J_{m''_n,\nu}(\theta_n)d^J_{m'_n,\nu}(\theta_{n+1})\Bigg]\\
&\times e^{-(i/2\alpha)(c_1+c_2)(\psi_n-\psi_{n+1})-(i/2\alpha)(c_1-c_2)(\varphi_n-\varphi_{n+1})}\\
&\times\begin{pmatrix} e^{-(i/2)(\varphi_n-\varphi_{n+1})} & 0\\ 0 & e^{+(i/2)(\varphi_n-\varphi_{n+1})}\end{pmatrix}. \qquad (58)
\end{aligned}$$

After substituting this result in (15), we will need to compute the following expression:

$$\Lambda^{\mp}(t_n)K^{\mp}(y_n,y_{n+1};t_n-t_{n+1}). \qquad (59)$$

This can be evaluated using the following results corresponding respectively to the derivation over the external sources at the instant s_n:

$$\begin{aligned}
&\frac{\delta}{\delta\pm(s_n)}I^{\Upsilon}\left(m''_n,m''_n;s_n-s_{n+1}\right)|_{\Upsilon=\Upsilon^*=0} \qquad (60)\\
&\to\frac{\delta}{\delta\Upsilon(s_n)}I^{\Upsilon}(m''_n,m''_n;s_n-s_{n+1}\,|_{\Upsilon=\Upsilon^*=0}
\end{aligned}$$

and

$$\begin{aligned}
&\frac{\delta}{\delta\pm^*(s_n)}I^{\Upsilon}\left(m''_n,m''_n;s_n-s_{n+1}\right)|_{\Upsilon=\Upsilon^*=0} \qquad (61)\\
&\to\frac{\delta}{\delta\Upsilon^*(s_n)}I^{\Upsilon}(m''_n,m''_n;s_n-s_{n+1})\,|_{\Upsilon=\Upsilon^*=0}\ .
\end{aligned}$$

The integration over the Euler variables is facilitated by using the following orthogonalization relations:

$$\int_0^{2\pi}d\varphi\int_0^{4\pi}d\psi e^{i(m-m')\varphi}\cdot e^{i(\nu-\nu')\psi}=8\pi^2\delta_{m,m'}\cdot\delta_{\nu,\nu'} \qquad (62)$$

and

$$\int_0^{\pi}d^J_{m,\nu}(\theta)\cdot d^J_{m',\nu'}(\theta)\sin\theta d\theta=\delta_{m,m'}\cdot\delta_{\nu,\nu'}\cdot\frac{1}{2J+1}. \qquad (63)$$

The Wigner function $d^J_{m,\nu}(\theta)$ vanishes for $J<\max\{|m|,|\nu|\}$, so we may shift the summation of (58) by letting $J=n+\max\{|(1/2\alpha)(c_1-c_2)|,|(1/2\alpha)(c_1+c_2)|\}=n+(1/2\alpha)(c_1+c_2)$, $(n=0,1,2,\cdots)$.

Therefore, the series of the propagator given by (15) will be reduced to

$$\begin{aligned}
K(y_b,y_a;T)=e^{-i\sigma_y(\kappa y_b)}&\Bigg[\alpha\left(\sin(2\alpha y_b)\sin(2\alpha y_a)\right)^{1/2}\\
&\times\sum_{n=0}^{\infty}(2n+(c_1+c_2)/\alpha+1)\exp(-iET)\\
&\times\begin{pmatrix}A(y_b,y_a;T) & B(y_b,y_a;T)\\ C(y_b,y_a;T) & D(y_b,y_a;T)\end{pmatrix}\Bigg]e^{i\sigma_y(\kappa y_a)}, \qquad (64)
\end{aligned}$$

where the elements of the matrix are given by the following expressions:

$$\begin{aligned}
&A(y_b,y_a;T) \qquad (65)\\
&=d^{n+(1/2\alpha)(c_1+c_2)}_{(1/2\alpha)(c_1-c_2)+(1/2),(1/2\alpha)(c_1+c_2)}(2\alpha y_b)\\
&\times d^{n+(1/2\alpha)(c_1+c_2)}_{(1/2\alpha)(c_1-c_2)+(1/2),(1/2\alpha)(c_1+c_2)}(2\alpha y_a)\\
&\times\left[1+\sum_{n=1}^{\infty}(\omega)^{2n}\int_0^T ds_1\int_0^{s_1}ds_2\cdots\int_0^{s_{2n-1}}ds_{2n}\right]
\end{aligned}$$

$$\begin{aligned}
&B(y_b,y_a;T) \qquad (66)\\
&=d^{n+(1/2\alpha)(c_1+c_2)}_{(1/2\alpha)(c_1-c_2)-(1/2),(1/2\alpha)(c_1+c_2)}(2\alpha y_b)\\
&\times d^{n+(1/2\alpha)(c_1+c_2)}_{(1/2\alpha)(c_1-c_2)+(1/2),(1/2\alpha)(c_1+c_2)}(2\alpha y_a)\\
&\times\left[\sum_{n=0}^{\infty}(i\omega)^{2n+1}\int_0^T ds_1\int_0^{s_1}ds_2\cdots\int_0^{s_{2n-1}}ds_{2n}\right]
\end{aligned}$$

$$\begin{aligned}
&C(y_b,y_a;T) \qquad (67)\\
&=d^{n+(1/2\alpha)(c_1+c_2)}_{(1/2\alpha)(c_1-c_2)-(1/2),(1/2\alpha)(c_1+c_2)}(2\alpha y_b)\\
&\times d^{n+(1/2\alpha)(c_1+c_2)}_{(1/2\alpha)(c_1-c_2)+(1/2),(1/2\alpha)(c_1+c_2)}(2\alpha y_a)\\
&\times\left[\sum_{n=0}^{\infty}(i\omega)^{2n+1}\int_0^T ds_1\int_0^{s_1}ds_2\cdots\int_0^{s_{2n-1}}ds_{2n}\right],
\end{aligned}$$

$$\begin{aligned}
&D(y_b,y_a;T) \qquad (68)\\
&=d^{n+(1/2\alpha)(c_1+c_2)}_{(1/2\alpha)(c_1-c_2)-(1/2),(1/2\alpha)(c_1+c_2)}(2\alpha y_b)\\
&\times d^{n+(1/2\alpha)(c_1+c_2)}_{(1/2\alpha)(c_1-c_2)-(1/2),(1/2\alpha)(c_1+c_2)}(2\alpha y_a)\\
&\times\left[1+\sum_{n=0}^{\infty}(i\omega)^{2n}\int_0^T ds_1\int_0^{s_1}ds_2\cdots\int_0^{s_{2n-1}}ds_{2n}\right],
\end{aligned}$$

with

$$\begin{aligned}
E&=\frac{\alpha^2}{2m}\left(2n+(c_1+c_2)/\alpha+1\right)^2\\
&\quad+\frac{\kappa^2}{2m}-\frac{1}{2m}(c_1-c_2)^2, \qquad (69)\\
\omega&=(2\kappa\alpha/m)\Big[(n+c_1/\alpha+1/2)\\
&\quad\times(n+c_2/\alpha+1/2)\Big]^{1/2}. \qquad (70)
\end{aligned}$$

It is also easy to show that

$$\left[1+\sum_{n=0}^{\infty}(\mathrm{i}\omega)^{2n}\int_0^T \mathrm{d}s_1\int_0^{s_1}\mathrm{d}s_2\cdots\int_0^{s_{2n-1}}\mathrm{d}s_{2n}\right] = \cos(\omega T) \tag{71}$$

and

$$\sum_{n=0}^{\infty}(\mathrm{i}\omega)^{2n+1}\int_0^T \mathrm{d}s_1\int_0^{s_1}\mathrm{d}s_2\cdots\int_0^{s_{2n-1}}\mathrm{d}s_{2n} = \mathrm{i}\sin(\omega T)\,. \tag{72}$$

Substituting all these results in (64), we will obtain the propagator written in its spectral decomposition form and we readily identify respectively the energy spectrum and the corresponding wave functions:

$$\begin{aligned} E_{\pm} = {} & \frac{\alpha^2}{2m}\left(2n+(c_1+c_2)/\alpha+1\right)^2 \\ & -\frac{1}{2m}(c_1-c_2)^2+\frac{\kappa^2}{2m} \\ & \pm(2\kappa\alpha/m)\left[(n+c_1/\alpha+1/2)(n+c_2/\alpha+1/2)\right]^{1/2} \end{aligned} \tag{73}$$

and

$$\begin{aligned} \mathbf{\Phi}_{+,n}(y) & \\ = {} & \left[\alpha\left(n+(c_1+c_2)/2\alpha+1/2\right)\sin(2\alpha y)\right]^{1/2}\mathrm{e}^{-\mathrm{i}\sigma_y(\kappa y)} \\ & \cdot\begin{pmatrix} d^{n+(1/2\alpha)(c_1+c_2)}_{(1/2\alpha)(c_1-c_2)+(1/2),(1/2\alpha)(c_1+c_2)}(2\alpha y) \\ d^{n+(1/2\alpha)(c_1+c_2)}_{(1/2\alpha)(c_1-c_2)-(1/2),(1/2\alpha)(c_1+c_2)}(2\alpha y) \end{pmatrix}, \end{aligned} \tag{74}$$

$$\begin{aligned} \mathbf{\Phi}_{-,n}(y) & \\ = {} & \left[\alpha\left(n+(c_1+c_2)/2\alpha+1/2\right)\sin(2\alpha y)\right]^{1/2}\mathrm{e}^{-\mathrm{i}\sigma_y(\kappa y)} \\ & \cdot\begin{pmatrix} -d^{n+(1/2\alpha)(c_1+c_2)}_{(1/2\alpha)(c_1-c_2)+(1/2),(1/2\alpha)(c_1+c_2)}(2\alpha y) \\ d^{n+(1/2\alpha)(c_1+c_2)}_{(1/2\alpha)(c_1-c_2)-(1/2),(1/2\alpha)(c_1+c_2)}(2\alpha y) \end{pmatrix}. \end{aligned} \tag{75}$$

These results agree exactly with those of the literature [5].

4 Conclusion

In the present paper we have calculated the explicit expression of the propagator relative to a neutral spinning particle in interaction with a linear increasing rotating magnetic field and a Poschl–Teller oscillator potential. This has been firstly treated using the factorization method and we have been able to reconsider it via path integral techniques. These exact calculations represent the first attempt to found a general method. To treat the spin dynamics, we have used the Schwinger recipe which replaces the Pauli matrices by a pair of fermionic oscillators. The introduction of a particular rotation has then simplified somewhat the Hamiltonian of the considered system. This modification contributes by an effective potential and couples the exterior velocity to the spin of the particle. To overcome this difficulty, we have introduced fermionic external current sources and have then reduced the problem to that of the spinning forced Poschl–Teller oscillator with the spin coupled to an external derivative source. As a consequence, we have been able to integrate over the spin variables described by fermionic oscillators and the result is given as a perturbation series. Next, to simplify the problem of the forced Poschl–Teller oscillator, we have converted it to the sphere via an extension of the dimension. Accordingly, the perturbation series is summed thanks to a Laplace transformation and the use of some recurrence formulae of the eigenfunctions of the angular momentum. We have also appropriately determined the energy spectrum and the corresponding wave functions.

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